Experimental solid mechanics: Towards the fourth dimension?

Eikology team

MATMECA Seminar, ONERA, Feb. 2016
Tomography

Image materials

Image large defects

Image Analysis

Segmentation
Tomography

Image materials

Image large defects

Image motion

Digital Volume Correlation
Tomography

Image materials

Image large defects

Image motion

Image small defects

Topological Difference

Digital Volume Correlation
Tomography

Image materials

Image large defects

Image motion

Image small defects

Image mechanical properties

Identification

Modelling

Digital Volume Correlation
Tomographic « distillation »

- Image materials
- Image large defects
- Image motion
- Image small defects
- Image mechanical properties

Big data

Redundancy exploited for validation

Data reduction from a priori knowledge

Key data
Outline

➢ Imaging materials and their kinematics from in situ testing

➢ From kinematics to mechanics

➢ Perspectives, challenges and suggestions

Warning: pro domo plea, but wide open for discussion
From material science to solid mechanics

IMAGING MATERIALS & THEIR KINEMATICS
Imaging in 3D

- A dream came true
- Essential for Material Science
- Key for industry (NDT & metrology)
- Unavoidable for experimental mechanics!

Wise investment into the future!
Tomography

For exceptional facilities (synchrotron beam light) …
Tomography

For exceptional facilities (synchrotron beam light) ...

... to lab scale equipment
Schematically

- Imaging based primarily on X-ray absorption

\[
I(r, \phi) = I_0(r) \exp \left( - \int_{\text{rayon}} f(x) dx \right)
\]

\[
s(r, \phi) = -\log \left( \frac{I(r, \phi)}{I_0(r)} \right) = \int_{\text{rayon}} f(x) dx
\]
Radiographs

- (colog)-radiographs

\[ s(\mathbf{r}, \phi) \]
Reconstruction

• Reconstruction

\[ f(\mathbf{x}) = \mathcal{R}[s(\mathbf{r}, \phi)] \]
\[ f = \mathcal{R}[s] \]

• Projection

\[ s(\mathbf{r}, \phi) = \mathcal{P}_\phi[f(\mathbf{x})] \]
\[ s = \mathcal{P}_\{\phi\}[f] \]
High-Tech Materials

Composite

Polymer foam
(double scale)
Low-Tech Materials

Gypsum

Stone wool

NG Cast Iron

Cement paste
Tomography

• Spectacular progress!
  – Spatial resolution
  – Temporal resolution
  – Wealth of information
  – Accessibility
  – Data analysis
Perspectives 1

• “Educated” tomographic reconstruction

• Exploitation of non voxel images (e.g. CAD based, reference based, or having a sparse parametrization)

• Instrument development: Phase contrast, or Diffraction contrast
In situ mechanical testing

• Severe constraints …
  … that can be overcome!
In situ mechanical testing

• **Tension-Compression**
  – Up to 1000 N
  – Displacement control
    • Axial displacement range of up to 15 mm
    • Displacement rate from 0.1 \( \mu \text{m/s} \) up to few mm/s

• **Sample**
  – Compression
    • \( \varphi_{\text{max}} = 10 \text{ mm} \)
    • \( h_{\text{max}} = 20 \text{ mm} \)
  – Traction
    • \( l = 2 \) to 3 mm
    • \( h_{\text{max}} = 20 \text{ mm} \)

In situ mechanical testing

- **Tension-Compression**
  - Up to 1000 N
  - Displacement control
    - Axial displacement range of up to 15 mm
    - Displacement rate from 0.1 μm/s up to few mm/s

- **Sample**
  - Compression
    - $\varphi_{\text{max}} = 10$ mm
    - $h_{\text{max}} = 20$ mm
  - Traction
    - $l = 2$ to 3 mm
    - $h_{\text{max}} = 20$ mm

In situ mechanical testing

- **Tension-Torsion-Compression**
  - Axial load up to ±20 kN;
  - Torque up to ±0.1 kNm
  - Displacement control
    - Axial displacement range of 50 mm
    - Displacement rates from 10 µm/min to 1.0 mm/min
    - Rotation range ±40°,

- **Sample**
  - Compression
    - \( \varphi_{\text{max}} = 50 \text{ mm} \)
    - \( h_{\text{max}} = 100 \text{ mm} \)
  - Tension
    - \( h_{\text{max}} = 120 \text{ mm} \)
This 100kg testing device fits elegantly into our tomograph …
… though somewhat tightly!
First in situ mechanical test

Oedometric compression on plaster foam
Digital Volume Correlation (DVC)

- Digital image correlation (DIC) in 3D
- Computing displacement field that accounts for the evolution in between two states imaged in similar conditions

\[ g(x + U(x)) = f(x) + \eta(x) \]

Global DIC/DVC

- Variational formulation (suited to white gaussian noise)

\[ U = \text{Arg min}_{U \in E} \| g(x + U(x)) - f(x) \|^2 \]

- Ill-posed inverse problem

- Nasty non-lineararities (the unknown is the argument of a rapidly varying function, the material texture)

Global DVC

Description of the kinematics

\[ \mathbf{v}(x) = v_i N_i(x) \]

- Our choice: Finite Element basis, or any other choice being mechanically legitimate
  - Continuous field (or more regular)
  - Easy interface with modelling
  - Benefit from novel FEM strategies (e.g. X-FEM, remeshing)
Global DVC

$U_x$

-9.3

-9.9

-10.5

$U_y$

9.8

9.2

8.6

$U_z$

-9.3

-10.3

-11.3
Displacement field

- Fatigue crack in nodular graphite cast iron
- C8 DVC

1 voxel $\leftrightarrow$ 3.5 $\mu$m

*[N. Limodin et al, Acta Mat. 57, 4090, (2009)]
DVC Residuals \[ |g(x + U(x)) - f(x)| \]

- Fatigue crack in nodular graphite cast iron
- C8 DVC

From crack geometry to extended DVC

Stress intensity factors

![Graph showing stress intensity factors](image)
Identification chain

*[
J. Rannou et al, CMAME 199, 1307-1325, (2010)]*
Perspectives 2
DVC and extensions

• Adaptative multiscale DVC
• Noise management
• Residual-based re-meshing
• Exploitation of “topological differences” for segmenting, measuring, identifying, detecting, …
Topological difference
New frontiers

BEYOND THE LIMITS
DVC as a medium

\[ \sigma = f(\varepsilon) \]

\[ \sigma < \sigma_c \quad K < K_c \]
DVC

• Principle:

\[ U = \operatorname{Arg\,min}_{U \in E} \| g(x + U(x)) - f(x) \|^2 \]

• The choice of the sub-space \( E \) is a nice way to add an additional information (regularization)
Two routes for regularization

• Strong regularization:
  – $\mathcal{E}$ is designed to have the smallest possible dimensionality
  – A small uncertainty is expected
  – It is however delicate to picture the most appropriate space $\mathcal{E}$ (not generic)
  – “Haute couture”
Example of strong regularization

Indentation of plasterboard

360,000 dof turned to 9

A. Bouterf et al, Strain 50, 444, (2014)
Correlation and identification

• Looking for material properties: \{ \lambda_i \}
• Computation of \( U \) and \( \partial U / \partial \lambda_i \)

\[ \frac{\partial}{\partial \lambda_i} \sum_i f_i(x + U + \frac{\partial U}{\partial \lambda_i} \Delta \lambda_i) - f \right\|^2 \]
Second route

• Weak regularization:
  – A penalty based on the distance from $U$ and its projection on $E$ is introduced
  – The choice of an appropriate metric is important

$$T_{reg}[U] = \min_{V \in E} \| U(x) - V(x) \|^2$$

– Alternatively, $E$ can be defined as the kernel of an operator acting on $U$
Al alloy specimen: equivalent strain field

T. Taillandier-Thomas, T. Mogeneyer, F. Hild
Al alloy specimen: equivalent strain field
Al alloy specimen: equivalent strain field

DVC only

Elastic regularization (isotropic and isochoric)

Elastic regularization anisotropic (tetragonal)
Perspectives 3

- New test design
- Multiphase identification
- Identification of complex constitutive laws
- Efficient computation
One additional challenge?

4D
DVC without reconstruction

• Another approach to DVC
• Can the number of projections be reduced?
• Kinematic complexity is much less than that of the microstructure
More efficient?

1000 projections
1000×1000 pix

Reconstruction

\( f(x) \)

10⁹ pix
More efficient?

1000 projections
1000×1000 pix

Reconstruction

\( f(x) \)

10^9 pix

Reconstruction

\( g(x) \)

10^9 pix

1000 projections
1000×1000 pix
More efficient?

1000 projections
1000×1000 pix

Reconstruction

$f(x)$
$10^9$ pix

$g(x)$
$10^9$ pix

Reconstruction

$U(x)$
3×10^6 data
DVC without reconstruction

1000 projections
1000×1000 pix

Reconstruction

\( f(x) \)

10⁻⁹ pix

\( \tilde{g}(x) \)

3×10⁶ unknowns

\( U(x) \)

3×10⁶ data
DVC without reconstruction

1000 projections
1000×1000 pix

10 projections
1000×1000 pix
=10^7 data

Reconstruction

f(x)
10^9 pix

Projection

g̃(x)
3×10^6 unknowns

3×10^6 data

U(x)
$N_{\text{rad}} = 600$
\[ N_{\text{rad}} = 48 \]
\[ N_{\text{rad}} = 24 \]
$N_{\text{rad}} = 12$
$N_{\text{rad}} = 6$ \hspace{1cm} U_z
$N_{\text{rad}} = 3 \quad U_z$
$N_{\text{rad}} = 2$ \quad \mathbf{U}_z
DVC without reconstruction

• Gain in acquisition time = 99.7 % !
• Above 300-fold gain in saving.
• Temporal resolution can be increased by a factor of 300!
• Can be combined with multiscale and elastic regularization.
Example of a fatigue crack

- Sample-based mesh
Example of a fatigue crack

- Displacement estimated from 2 projections
Example of a fatigue crack

- Projection residuals
DVC without reconstruction

Bonus:
- A proper handling of noise now becomes easily feasible
Perspectives 4

• 4D Dynamic Tomography
• Cone-beam geometry is to be implemented
• Adaptation to noise
• Application to identification
• More complex evolutions than motion
Perspectives

• Tomography allows for the analysis of materials in their mechanical expression (in situ tests / reconstruction / DVC / identification)
• Coupling different treatments increases robustness and provide data-sober solutions
• Considerable savings can be achieved
• Today tomography is fantastic, and tomorrow will be even more so!
Tomographic « distillation »

Image materials
Image large defects
Image motion
Image small defects
Image mechanical properties

Big data
Data reduction from a priori knowledge
Redundancy exploited for validation
Key data
Experimental solid mechanics: Towards the fourth dimension!

Eikology team

MATMECA Seminar, ONERA, Feb. 2016
Perspectives 1

• “Educated” tomographic reconstruction

• Exploitation of non voxel images (e.g. CAD based, reference based, or having a sparse parametrization)

• Instrument development: Phase contrast, or Diffraction contrast
Perspectives 2
DVC and extensions

• Adaptative multiscale DVC
• Noise management
• Residual-based re-meshing
• Exploitation of “topological differences” for segmenting, measuring, identifying, detecting, …
Perspectives 3

• New test design
• Multiphase identification
• Identification of complex constitutive laws
• Efficient computation
Perspectives 4

- 4D Dynamic Tomography
- Cone-beam geometry is to be implemented
- Adaptation to noise
- Application to identification
- More complex evolutions than motion